DESIGN OF FACTORIAL EXPERIMENTS IN SMALL BLOCKS

by

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ABSTRACT

The problem this study attempts to answer may be briefly described as follows. How can a factorial experiment be designed in a simple manner making use of whatever available blocks (which may or may not allow equal replication of treatments) so as to be able to estimate the important contrasts with a pre-specified order of precision?

The inverse variance covariance matrix $\underline{\Omega}^{-1}$ for contrast effects (contrasts defined to be orthogonal to each other) obtained using least squares equations defines an important matrix which determines the block - contrast, and hence contrast - contrast non orthogonalities. This matrix is obtained by multiplying the matrix defining the set of contrasts $\underline{w}$, and the incidence matrix of the design, $\underline{n}$, and is termed the normalised basic score matrix, $\underline{s}$. ($\underline{s} = \underline{w}.\underline{n}$).

A zero basic score corresponding to a contrast and a block indicates that this contrast is orthogonal to the block. On the other hand, a higher (absolute) normalised basic score indicates a higher block - contrast non orthogonality.

The component $\underline{s}\underline{k}^{-\delta}\underline{s}'$ of the diagonal element of $\underline{\Omega}^{-1}$ corresponding to a contrast defines the reduction of direct replication of a contrast due to blocking. This is termed 'the score for contrast e', and denoted by ss_e.

It appears that making ss_e small for a given contrast e ensures a higher precision for this contrast provided that the contrast is sufficiently replicated. However, this quantity can not be made small for all the contrast effects, since over a complete set of orthogonal contrasts, the sum of the contrast scores is a constant.

Hence, the philosophy used in this study to obtain the desired order of precision for the contrasts is to allocate treatments to blocks so as to obtain the values for $(r_e - ss_e)$ for each contrast to be in the same order as the order of importance of contrasts, as far as the blocks permit it. Since however this does not take in to account the off- diagonal terms of $\underline{\Omega}^{-1}$, the treatments need to be further re-arranged among blocks to obtain smaller values for the cross-product terms among contrasts in the basic score matrix (weighted by reciprocal of block sizes), $\sum_i (s_{ei} s_{e'i} / k_i)$, without disturbing the order acquired for the values of $(r_e - ss_e)$ for each contrast.

The procedure is simple and was found to be able to produce efficient small factorial designs. To assess its effectiveness in dealing with constructing large factorial designs the philosophy is translated in to a computer algorithm. Using this, it was found that the above method is capable of dealing with large factorials as well.

A brief study of constructing row and column designs using a similar principle is also included.