Mathematical structure for accumulations given by integration

N.C. Ganegoda[1], S.K. Boralugoda[2], K.K.W.A.S. Kumara[1]
[1] Department of Mathematics, University of Sri Jayewardenepura
[2] Department of Mathematics, University of Colombo

Abstract

The structure presented here is very much applicable in ignoring unexpected variations that would hamper identifying general behavior corresponding to an accumulation. Underpinning concept prevails in "almost everywhere" fact, which associates with Lebesgue measure. The structural context is in fundamental nature, where further mathematical theories on curve fitting, cardinality of sets, non-smooth analysis and interval partitioning can be incorporated.

1. Introduction

A definite integral represents a sort of accumulation of a function. Quantitative matters concentrated over time, distance, angle etc. are modeled using integration in many applications [1]. Rapid or unexpected variations or impossibility of quantifying at some portions in the domain may curtail the applicability of smooth functions. Our structure restricts the burden of non-smooth situations via the condition of almost everywhere (a.e.) in measure theory.

2. Mathematical Preliminaries

In the context of measure theory, a property is said to be satisfied a.e. on a set X, if the set Y of all points in X that do not satisfy the property has measure zero [2]. In our approach, Lebesgue integration is preferred over Reimann integration since it has wider range of applicability. Simply the Lebesgue measure of an interval is its length. The fact that Lebesgue measure of a countable set of singletons is zero provides the basic approach for a.e. notion in our structural design.

2.1 Segment-into-point (SIP) reduction

Condition of a.e. is very much applicable in point-wise representation and a way of reducing function segments into a point-wise representation with a criterion to recognize unexpected variation has been presented below.

For instance, suppose a function f consists of three segments f_1, g and f_2 with constant slopes in each segment (Figure 1). Suppose the segment g represents an unexpected variation.

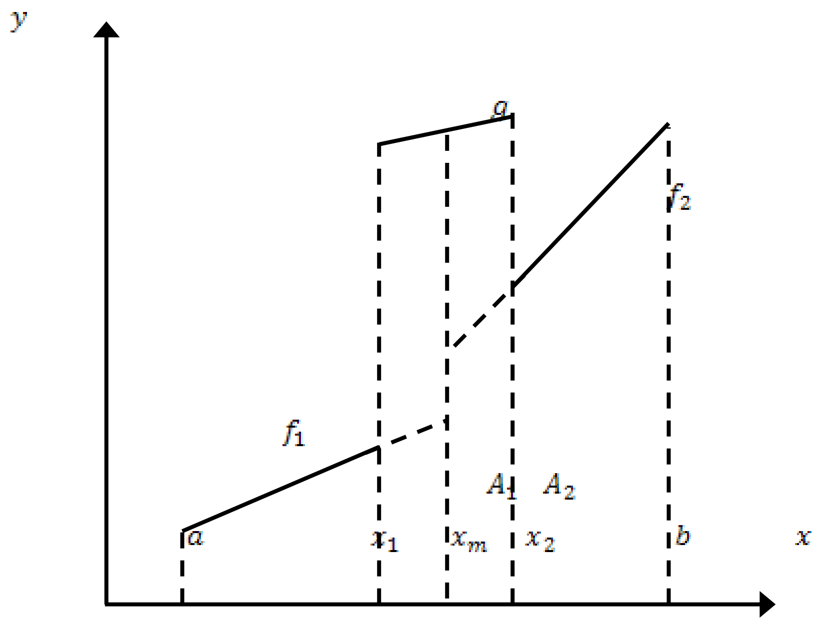


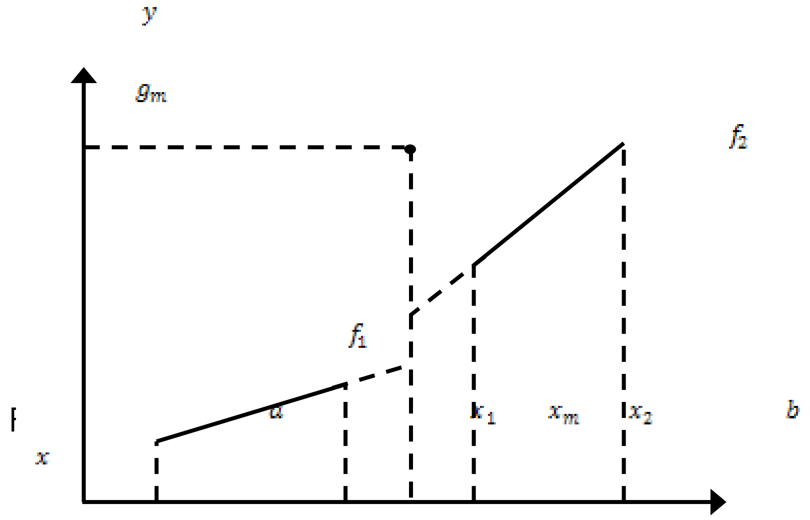
Figure 1: Function f with segments f_1, g and f_2

Now the additional accumulation due to g is calculated as $S = \left|\int_{x_1}^{x_2} g\,dx - A_1 - A_2\right|$. Here, A_1 is the integration of extended segment f_1 over (x_1, x_m) and A_2 is the integration of extended segment f_2 over (x_m, x_2). According to the modeled scenario x_m can be shifted between x_1 or x_2. If f_1 does not have constant slope, suitable segment extension should be made. The similar context is applicable for segment f_2 as well.

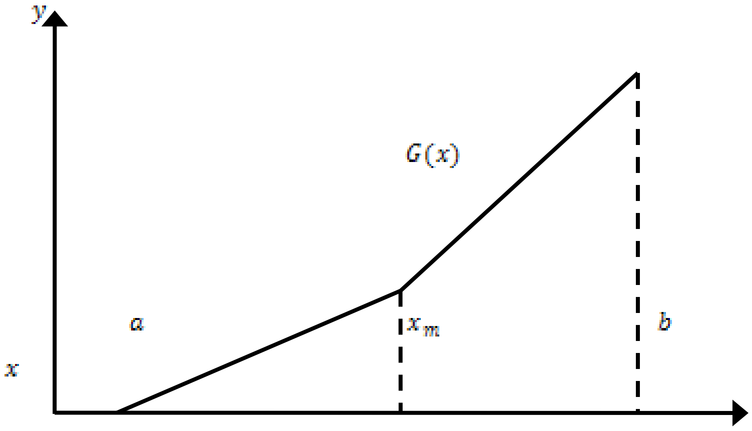
Now, an upper bound (u) can be used for additional accumulation S, where S is considered as unexpected when $S \le u$. However, a restriction could be applied for interval length $x_2 - x_1$, since smaller additional accumulation persisting for longer domain segments would not be negligible in some phenomena. Resultant SIP reduced function for the case in Figure 1 has been depicted in Figure 2. The function value at x_m can be averaged as $g_m = \int_{x_1}^{x_2} g\,dx / (x_2 - x_1)$ to indicate the variation occurs otherwise.

Once additional accumulation is neglected, accumulation G of f is calculated as follows.

$$G(x) = \begin{cases} \int_a^x f_1(x)dx & ; x \le x_m \\ \int_a^{x_m} f_1(x)dx + \int_{x_m}^x f_2(x)dx & ; x > x_m \end{cases}$$



Now, it is evident that variation due to g has been reduced just to one non-smooth point at x_m as depicted in Figure 3. It allows easier smoothing of $G(x)$ by curve fitting or interpolating technique.



d function of f

For a non-negative function f, the simplest case of having $\int f = 0$ is $f = 0$. But, a.e. approach simply expanding the context of f, where $\int f = 0$ for all $f = 0$ a.e. functions. On the other hand, if $\int_a^x f(\tau)d\tau = 0$ for all $x \in [a, b]$ for a Lebesgue integrable function f on $[a, b]$, then $f = 0$ a.e. [2].

Cardinality differences between countable and uncountable sets of domain points can be incorporated to formulate reliable interpretation on choosing f for modeling accumulation.

2.3 Smoothness of non-zero accumulations:

Obviously, $G(x) = \int_a^x f(\tau)d\tau$ is smooth when f is smooth. Though f loses smoothness at a point just because of losing continuity as a result of a different function value to limit at that point, then G does not lose smoothness at that point. If this happens at a set with measure zero, then a.e. smooth functions can be used to obtain smooth accumulations.

2.4 Unit integral behavior of non-zero accumulations:

Suppose $[a, b]$ is partitioned as $a = a_0, a_1, \dots, a_n = b$ and f_i denotes the function segment for i^{th} partition $[a_{i-1}, a_i]$ of a function f. Then the unit integral for i^{th} partition is defined by $G_i = \frac{\int f_i}{h_i}$, where $h_i = a_i - a_{i-1}$ and the integration is taken over $[a_{i-1}, a_i]$. There are straightforward lower and upper bounds for A_i in terms of supremum of f_i $(sup\, f_i)$ and infimum of f_i $(inf\, f_i)$ as presented below.

$$h_i inf\, f_i \le \int f_i \le h_i sup\, f_i \quad \Rightarrow \quad inf\, f_i \le G_i \le sup\, f_i$$

These sequential boundaries are important in modeling accumulation especially for functions with increasing and decreasing trends. The condition of a.e. further strengthens this claim.

3. Discussion

Notion of a.e. is much applicable when there is something unnecessary to be neglected. The structure that Lebesgue measure is defined upon provides this applicability through sets of measure zero. Distinguishing zero and non-zero accumulation would be a basic step, where one can improve with more categorization.

Many theories can be utilized upon our structure. For instance, SIP reduction allows smoother view on accumulation providing more exposure to curve fitting techniques. Furthermore, approximated derivatives at non-smooth points in accumulation allow illustrating possible behavior in original function subjected to integration.

Reliability of data and assumptions claims how far we can proceed by incorporating experimental facts towards model and model towards predictions. In that sense, designing a general structure for accumulation would require modifications for a specific application.

References

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A detailed analysis of a genotype-environment study on rice in Sri Lanka

W.M.P.M. Wickramasinghe[1], W.N. Wickremasinghe[2] and R.A.B. Abeygunawardena[2]

[1] Department of Mathematical Sciences, Wayamba University, Kuliyapitiya

[2] Department of Statistics, University of Colombo, Colombo 3

Abstract

Genotype × Environment (variety × location) interactions are very important in variety trials repeated over different locations. The presence of interaction suggests that recommendation of varieties must be done according to their adaptability and stability over diverse conditions. Published research on Genotype - environment studies on rice in Sri Lanka are relatively few. This study focuses on a detailed yield analysis for the seasons Yala and Maha of a local varietal trial of ten 3 ½ -month rice varieties repeated over 7 locations and recommendations for cultivation are given based on the results of a combined analysis and subsequent measures.

1. Introduction

Variety × location interactions [1] usually occur in variety (genotype) trials repeated in different locations (environments). If there is no interaction, then varieties can be recommended uniformly over locations. However, if there is interaction, then the recommendations must be given according to their adaptability and stability over diverse environments.