ESTIMATING RELIABILITY AND PREDICTING WARRANTY OF AN ELECTRONIC DEVICE WITH MULTIPLE FAILURE MODES

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This research considers electronic devices which are prone to fail after some period of time. Electronic devices usually can have more than one cause of failure. In practice, it is important to differentiate between these and find the impact of different causes of failure. Differentiating between failure modes is required for improving reliability, for determining the cause of failure and to set warranty for the device. The data set consists of failure records of an electronic device. Due to confidentiality of the data neither the device name nor the description of the failure modes can be divulged. The time variable is measured in terms of cycles to failure. Here, the electronic device has several modes of failure which are categorized into three primary modes of failure with the less occurring failure modes being categorized as "other" failure mode. The primary objective of this study is to illustrate the use of reliability analysis for improving the reliability of an electronic device. In the analysis the device was considered as a series system in which the device fails when any one of the failure modes occur. Selected parametric models for each failure mode were combined to determine the reliability of the electronic device. The methods illustrated here have wide applicability. These can be applied to any electronic device such as computers, mobile phones, calculators etc., to electrical devices such as light bulbs, refrigerators, air conditioners, etc. to mechanical devices such as motor vehicles, air crafts and, military equipment etc.

***Key words*:** Reliability, Electronic device, log-logistic model, warranty

1. INTRODUCTION

The failure of a product or a device can occur due to different types of failures that are associated with the device of interest. When analyzing the failure times of such devices, it is important to take into account the distinct failure mode that causes the overall device to fail to gain an in-depth understanding of the failure pattern of such devices. Such analysis would enable to identify the weight or the effect that each failure mode carries to the overall failure of the device.

This study is aimed at analyzing failure data of an electronic device that consist of four failure modes such that the occurrence of one failure mode leads the whole device to fail. The literature consists of many examples of electronic devices where the occurrence of one failure mode will lead to the failure of the entire device and where the failure modes are independent. These devices/systems are known as series devices/systems. Authors such as Pham and Turkkan [1], Turkkan and Pham-Gia [2], Martz and Baggerley [3], Rajgopal and Mazumda [4] can be cited in this regard.

2. METHODOLOGY

2.1. Procedure for Analysis

In the literature, there exist basically two schools of methodologies for analyzing failure time data with multiple failure modes; namely, Engineering approach and the Statistical approach. The statistical approach is used to model different failure modes using suitable distributions and then to use that model for measuring the reliability of the device and to make predictions about the failure process.(Sozer et al. [5]; Chen [6] Tolio et al.[7]; Zhang et al. [8].

In line with the Statistical approach which is the approach undertaken in this study, suitability of a parametric distribution to accommodate each failure mode was tested. Probability plots (Meeker and Escobar [9]) were used to check the adequacy of each distribution considered and to select the most appropriate distribution for each distinct failure mode. Departures from a straight line in the probability plot suggests a lack of fit of the assumed distribution. Additionally simultaneous confidence bands (Meeker and Escobar [9]) were taken on the probability plot to quantify the magnitude of observed departures from the fitted parametric model. Lack of fit is strongly indicated if the departures from the straight line are beyond these confidence bands.

Once the suitable distributions are identified for each failure mode, separate survival regression models are fitted for four failure modes and the parameter estimates are obtained for each failure mode.

The survival function of Log-logistic distribution is:

$$S(t) = \frac{1}{1 + exp\left(\frac{\log t - \mu}{\sigma}\right)} \dots\dots\dots\dots\dots\dots\dots\dots (1)$$

The survival function of log-normal distribution is:

$$S(t) = 1 - \emptyset_{nor}\left(\frac{logt - \mu}{\sigma}\right) \dots\dots\dots\dots. (2)$$

Since the electronic device considered in this study is of the type of a series system, the survival function of the whole device is represented by the product of the survival functions of the individual failure modes. Thus, the survival function of the whole device takes the form

$$S(t) = \prod_{i=1}^{n} S_{(i)}(t) \dots\dots\dots\dots\dots\dots\dots\dots\dots (3)$$

Then, the cumulative failure probability of a series system can be regarded as:

$$F(t) = 1 - S(t) = 1 - \prod_{i=1}^{n} S_i(t) \dots\dots\dots (4)$$

In addition to point estimations of failure probabilities that can be calculated for the device using eq.(04), confidence limits for failure probabilities is also calculated to quantify the uncertainty in point estimates. The limits of confidence interval calculated for failure probability of the device can be considered as more accurate estimates for the failure probability of the device.

The logit transformed 100(1-α) % Confidence Interval for $F_i(t)$ (Meeker and Escobar [9]) is,

$$\left[\frac{\hat{F}(t)}{\hat{F}(t) + \left(1 - \hat{F}(t)\right) * w} \quad \frac{\hat{F}(t)}{\hat{F}(t) + \left(1 - \hat{F}(t)\right)/w}\right] (5)$$

$$where\ w = \exp\left(\frac{Z_{(1-\alpha/2)}\ S.E(\hat{F}(t))}{\hat{F}(t) * (1 - \hat{F}(t))}\right). (6)$$

Since it is not possible to obtain $S.E(\hat{F}(t))$ **d**irectly from software, Delta Method (Meeker and Escobar [9]) was used to calculate it manually.

3. RESULTS

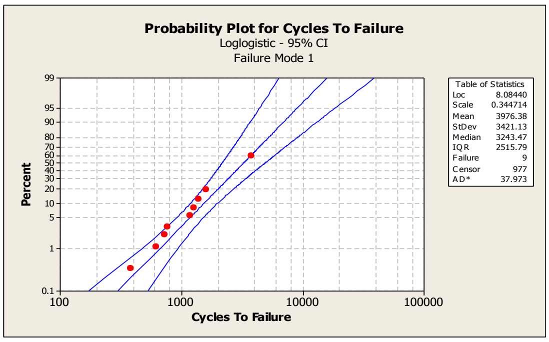
The data set considered had information on 986 electronic devices. The failure time was measured in terms of number of cycles taken at the time of failure and for those un-failed devices the number of cycles at the last inspection was recoded as their censored failure time. All together there were 74 failures recoded where as the cause of failure being mode 1for 9 devices, failure mode 2 for 37 devices, failure mode 3 for 22 devices, and 'other' failure mode for the rest of the 6 devices failed. Table 1 contains information on failures observed under each failure mode.

Table 1: Details of the Observed Failures

Failure Mode	No. Of failures	Minimum No of Cycles to fail	Maximum No of Cycles to fail
Mode 1	9	386	3770
Mode 2	37	2	2142
Mode 3	22	1	688
Other	6	68	699

Form the Table 1, it can be seen that Mode 2 and 3 have shown higher number of failures than the rest of the failure modes and also have shown very early failure times in comparison. Table 1 clearly indicates the effect each failure mode is not similar among failure modes in causing the device to fail. Therefore, this signals the importance of taking the mode of failure when analyzing the failure pattern of the device.

Through the probability plots drawn for the four failure modes (figure1), it was identified that Log-logistic distribution is the best suited distribution for the failure modes 1 and 2, and log- normal distribution was suitable for the failure modes 3 and "other". Then, regression models were fitted separately for the four failure modes, and parameter estimate were obtained for the four failure modes. (Please refer table 2)



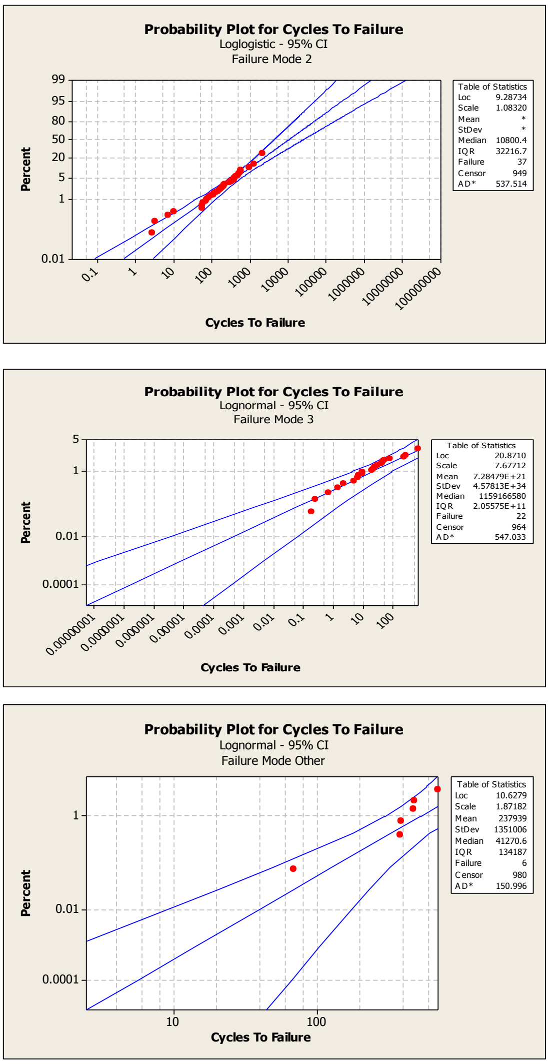


Figure 1 Probability plots for failure modes

Using the parameters estimates and the eq.(03), the reliability (survival) equation for the whole device can be written as:

$$s(t) = \frac{1}{1+exp\left(\frac{logt-8.08}{0.34}\right)} * \frac{1}{1+exp\left(\frac{logt-9.3}{1.08}\right)} * 1-\Phi_{nor}\left(\frac{logt-16.87}{3.01}\right) * 1 - \Phi_{nor}\left(\frac{logt-9.59}{0.71}\right) ..(7)$$

Table 2: Details of the Models fitted

Failure Mode	Distribution	Parameter Estimates	Std. Errors
Mode 1	Log-Logistic	$\mu_1=8.08$ $\sigma_1=0.34$	0.2072 0.0609
Mode 2	Log-Logistic	$\mu_2=9.29$ $\sigma_2=1.08$	0.4491 0.1396
Mode 3	Log-normal	$\mu_3=16.87$ $\sigma_3=3.01$	2.3582 .6064
Other	Log-Normal	$\mu_4=9.59$ $\sigma_4=.71$	.8705 0.1897

Then, the reliability of the system can be calculated in terms of probability of surviving beyond a specific number of cycles of usage of the device. This can also be used to predict the warranty.
The failure probability of the device can be obtained by substituting eq.(07) into the relationship given in eq.(04). The failure probabilities are calculated for every 500th cycles of usage and are given in table 3.

Table 3: Reliability Estimates for the Device

Cycles of Usage	Probability of Failure				
	Mode 1	Mode 2	Mode 3	Other	Device
500	0.004	0.05	0.0002	9.9e-07	.06
1000	0.03	0.1	0.0005	7.9e-05	.13
1500	0.09	0.14	0.0007	0.00067	.22
2000	0.19	0.17	0.001	0.0025	.34

When considering the failure probabilities in table 3, it can be clearly seen that failure pattern in terms of probability differs among failure modes. Probability of a failure under mode 1 and 2 are higher form the 500 cycles onwards then other two failure modes. That is failure modes 1 and 2 shows a high probability of occurrence even at early stages of usage. Where as failure mode 3 and 4 show a less probability of failure throughout the usage. The last column gives the probability of failure of the overall device that can result from the occurrence of any of the failure mode. There it can be seen that failure probability is increasing gradually with the usage. In setting warranty periods it is actually a cut-off of this usage that is to decided to bare the cost of repair/replacement of the devices that fail by that period.
It is to be noted that about 13% of the devices gets entitled for warranty if the warranty is set for 1000 cycles of usage, about 34% of the devices are entitle for warranty if the warranty period is set to 2000 cycles of usage and so on. Likewise, the manufactures of these devices can make a proper estimate of the warranty period taking into account the percentage of devices that get entitle for warranty at different usages.
As mentioned, since it is not conclusive to rely only on point estimates of the failure probabilities it is important to obtain confidence limits of the failure probabilities calculated above. Therefore, confidence limits of the failure probability were calculated by using eq.(05 and 06).
Table 4 contains 95% confidence limits of the each failure probabilities calculated for the device. Rather than a point estimate for the

percentage of failures occurring at different cycles of the device usage, confidence limits gives a better insight for the possibility of failure in the usage of the device. The decision of the warranty period (i.e cycles of usage that gives the consumer a warranty) is a compromise between the percentage of failures that can occur by that usage and the cost of replacing/repairing the devices claimed for warranty by that period.

Table 4: Confidence Intervals for the Failure Process

Cycles of Usage	Probability of failing	Confidence Intervals	
		Lower Limit	Upper Limit
500	0.06	0.054	0.064
1000	0.13	0.11	0.15
1500	0.22	0.18	0.27
2000	0.34	0.24	0.46

4. CONCLUSIONS

The main objective of this study was to analyze failure data of a device taking into account the mode of the failure also. Examining the number of failures observed under each failure mode and the timing of the failure with respect to each failure mode reveled that there is a clear distinction among failure patterns of each failure mode. Suitable parametric distributions were then selected for each failure mode separately and the parameter estimates of the models were obtained. Those models were combined to obtain estimates for the reliability of the device as whole in which the device was considered as of series system.

The estimates obtained for failure probability of the device at different usages can used as a basis in deciding the warranty period in terms of cycles of usage. For devices that are not used continuously over time, the use of a time scale like 'cycles of usage' can capture the failure pattern more precisely as device tend to fail with its usage rather than the calendar period since it had been manufactured.

More over, if the manufacture have a specific percentage of failure that they can manage for their warranty it is possible to get the exact value of the cycles of usage with respect to the failure percentage that the manufacture is willing to accommodate. For that inverse function of the eq.(07) can be used.

In conclusion, it can be mentioned that this study revealed the importance of taking into account the mode of failure when analyzing the failure data of device with multiple failure modes. Further, it explained a statistical methodology for determining warranty periods according to the percentage of failures that corresponds with the warranty period.

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