Modeling Daily Rainfall using Markov Chains

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Background

A number of authors have used Markov chains to model the daily occurrence of rainfall. After the work of Gabriel and Neumann (Gabriel and Neumann,1962) who applied the Markov chain model successfully to describe Tel Aviv daily rainfall data, a number of researchers have applied a similar technique to study rainfall in widely different geographical regions. However, except for a few early studies (Weerasinghe, 1989, Punyawardena and Kulasiri 1998), not much work has been carried out to model the wet and dry spell sequence of daily rainfall observed in Sri Lanka. The main objective of this work is to use Markov chains to study the wet and dry spells of observed at the Colombo meteorological station (1941-2000) based on daily precipitation.

In the first order Markov chain probability model, the probability of rain occurring on a given day (wet day) depends solely on the condition of the previous day. The two transitional probabilities can be defined as p_0 and $(1-p_1)$ where p_0 is the probability of a wet day given the previous day is dry, and $(1-p_1)$ is the probability of a dry day, given the previous day is wet.

The probability of a dry spell of length *n* can be defined as *n* successive dry days followed by a wet day. Similarly, the probability of a wet spell of length *n* can be defined as *n* successive wet days followed by a dry day. These can be shown as;

$p_0(1-p_0)^{n-1}$ and $(1-p_1){p_1}^{n-1}$

The mean lengths of dry and wet spells are;

$L_d = 1/p_0$ and $L_w = 1/(1-p_1)$

Although the first order model can be extended to develop higher order models, several past studies (Punyawardena and Kulasiri 1998) have shown that the first order model is sufficient for this region and that there is no advantage in using higher models. Thus, the analysis and comparisons were carried out with the results derived from the first order Markov model.

Analysis of wet and dry spells

Variation of transition probabilities

Transition probabilities show considerable variation throughout the year. The probability of observing a wet day followed by a wet day shows a bimodal pattern of rainfall occurrence with two distinct peaks with the first peak in May coinciding with the start of the southwest monsoon rains and the second peak in October coinciding with the start of the second inter-monsoon season. The probability of observing a dry day followed by a dry day is high during the northeast monsoon period and during the latter part of the southwest monsoon period. Thus, one can expect longer durations of wet spells during the months of May and October and longer durations of dry spells during the months of January and August.

Modeling sequences of wet and dry days

The accuracy of the Markov model in estimating sequences of wet and dry days is shown in Figure 1 for the Colombo weather station. The data and the model estimates agree reasonably well. For the southwest monsoon season (May to September) and the 2nd inter-monsoon season (October to November), a slight reduction in accuracies is seen in the estimate of the number of isolated one day wet spells (not shown here). The number of dry spells of one day duration is under estimated by the model for all seasons. The data and the model estimates agree reasonably well for dry spells of duration greater than 1 day. Perhaps higher order models may be able to fit the data better than 1st order model when 1 day spells are considered.

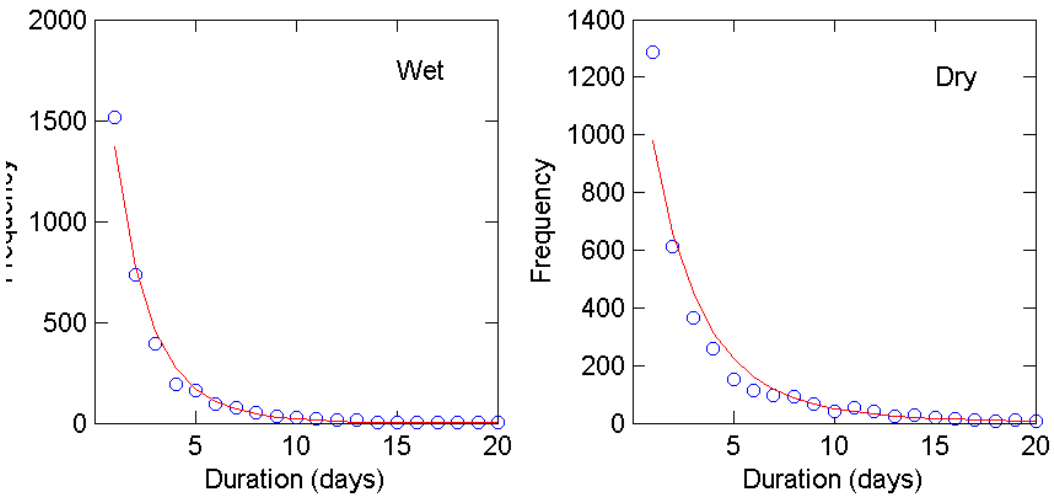

Figure 1: Comparison of observed (data) and estimated (line) sequence of wet and dry days.

Mean length of wet and dry spells

The mean lengths of wet and dry spells for each month predicted by the model were compared with the estimated values from the data set (see Figure 2). The mean annual duration of wet spells is 2.6 days while the mean annual duration of dry spells is 3.8 days.

As expected, the months receiving higher rainfall show longer durations of wet spells. In general, the agreement between the model prediction and estimates was quite reasonable. The data shows greater disagreement for the months having longer dry spells.

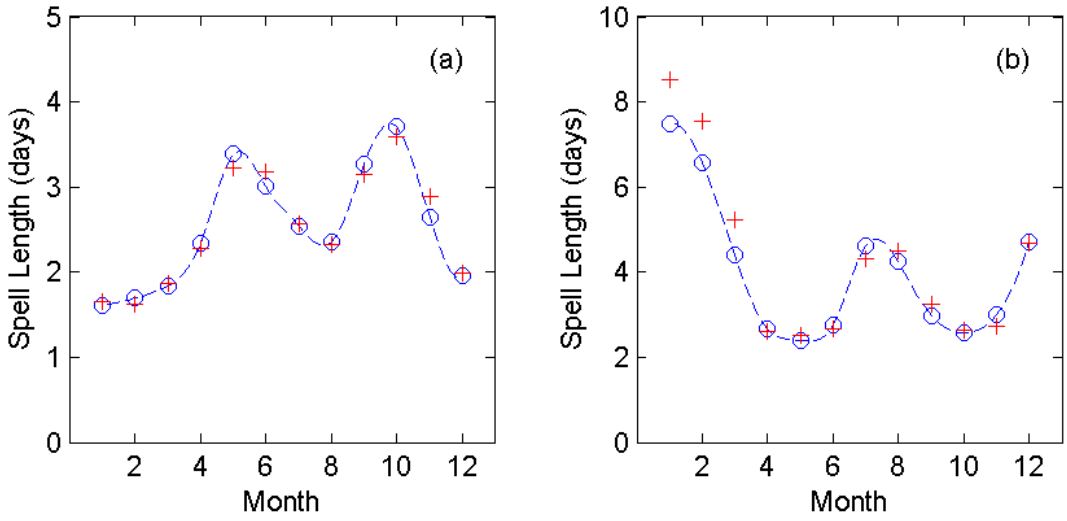

Figure 2: Simulated (+) and observed (o) average lengths of spells (a) Wet (b) Dry

Conclusions

This study focused on comparing the predictions of a model based on Markov chains and actual data on sequences of wet and dry spells and the mean lengths of wet and dry spells per month. Except for isolated single wet or dry events, the model is shown to describe the observed sequences of wet and dry spells which vary depending on the season. The mean length of wet spells calculated by the model and estimated by the data are accurate to within ±0.01 days.

References

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