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Simulation of lightning discharges: Influence of ground objects and positive leaders

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ABSTRACT

Cloud-to-ground (CG) lightning discharges in 3D domain were simulated using a stochastic dielectric breakdown model. The influence of ground objects on simulated lightning flashes was studied by introducing additional boundary conditions to the ground plane. It was observed that pointed structures on the ground have a higher probability of attracting simulated lightning discharges. An extension was introduced to the dielectric breakdown model to simulate the development of positive leaders that occur during CG lightning flashes. It was found that the height of the stepped leader tip above the ground (at the time when the positive leader initiation occurs) is dependent on the local electric field associated with the discharge step controlled by the parameter E_{break}. The height to the leader tip was found to decrease exponentially as E_{break} is increased.

1. INTRODUCTION

An electrical discharge is a common natural phenomenon that occurs during the dielectric breakdown of all gaseous, liquid and solid dielectrics. Various categories of discharges such as lightning flashes, surface discharges, and treeing in polymers occur as trajectories of luminous filaments, which often break into complex branched patterns. Although there are fundamental differences between underlying mechanisms of various discharge types, structure of branched tree-like discharge patterns often shows a close structural similarity within a large variety of discharge types.

In 1984, Niemeyer *et al* [1] introduced *Dielectric Breakdown Model* (*DBM or DB Model*) to explain the geometric structure of surface discharges in compressed SF_6 gas. The basic assumptions of their stochastic model are; the discharge pattern is equipotential (that is the voltage gradient along the structure is zero), and the probability of growth (p) at any point of the discharge is proportional to a power (η) of the local electric field (E) at that point. Niemeyer et al. studied their model via computer simulations in a 2D square lattice under 2D Laplace field, and obtained highly branched structures similar to experimental surface discharge patterns when the power (η) is set to 1.

In 1995, Sando et al. [2] extended the two dimensional DB model to simulate lightning discharges in three dimensions. They studied the changes in the geometric structure of the discharges with varying η and obtained discharge patterns that resemble actual lightning photographs when $\eta \approx 6$.

This paper presents results of a study conducted to simulate cloud-to-ground (CG) lightning discharge patterns in 3D using DB model. Main focuses of the study are;

- Investigating the influence of ground objects on simulated lightning flashes by introducing additional boundary conditions to the DB Model.
- Introducing an extension to DB model in order to simulate the formation of positive leaders which occur during lightning flash initiation.

2. METHODOLOGY

2.1. Dielectric breakdown model

DB process is defined in discrete coordinate space. Figure 1 shows the cross section of a sample 3D square lattice (in its initial state) used to produce lightning discharge patterns. Discharge pattern is indicated by black circles connected with thick lines. Dashed lines which connect each discharge point (black circle) with a neighbouring charge-free lattice point (white circle) represent possible breakdown links.

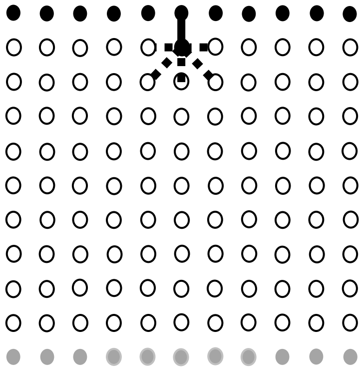

Figure 1: Initial charge configuration

Each lattice point has an electric potential value ϕ associated with it. The potential of the lattice points in the upper boundary of the lattice (which simulates the cloud base) are fixed to $\phi = 0$, while lattice points in the lower boundary (which simulates the ground plane) are fixed to $\phi = 1$.

Potential of the lattice points constituting the discharge pattern are also fixed to $\phi = 0$. As for the remaining charge-free lattice points (white circles), potential is defined by the Laplace equation;

$$\nabla\phi = 0 \ldots\ldots\ldots\ldots\ldots 1$$

DBM is an iterative process. The process starts with the initial boundary conditions consist of the negative cloud base and the positive ground plane. During each iteration of the algorithm, a new dashed line segment is chosen and added to the discharge pattern, linking a lattice point of the pattern with a new point. A new dashed line is chosen according to the weighted probability function given in equation 3 as explained below.

Let P denote a lattice point connected to the discharge pattern (black circle) while Q represents an adjacent charge-free lattice point (white circle). Magnitude of the component of electric field vector (local electric field) at P pointing in the direction of PQ is approximated by;

$$E_{PQ} = \frac{|\phi_P - \phi_Q|}{d} \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

Where d is the length of a dashed line segment. (For simplicity, d is set to 1).

Probability of choosing a particular *PQ* dashed line segment is given by;

$$p_{PQ} = \frac{(E_{PQ})^{\eta}}{\sum_{i=1}^{n} (E_{(PQ)_i})^{\eta}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

where the sum in the denominator refers to all possible breakdown links extended from discharge point *P*. With the selection of new discharge link, the potential of the newly added lattice point is fixed to $\phi = 0$ and it becomes part of the boundary conditions. Therefore, potential at charge-free lattice points have to be recalculated under the new boundary conditions by solving the discrete form of Laplace equation;

$$\begin{aligned}\phi_{\mathrm{i,j,k}} = \frac{1}{6}(&\phi_{\mathrm{i+1,j,k}} + \phi_{\mathrm{i-1,j,k}} + \phi_{\mathrm{i,j+1,k}} + \\ &\phi_{\mathrm{i,j-1,k}} + \phi_{\mathrm{i,j,k+1}} + \phi_{\mathrm{i,j,k-1}})\end{aligned} \quad \ldots\ldots\ldots\ldots\ldots\ldots 2$$

where i, j, k represent discrete lattice coordinates. To increase the speed of convergence, the system of equations generated by equation 4 was solved by the iterative technique *successive over-relaxation* [3].

2.2. Simulating positive leader growth

During real lightning flash initiation, when the stepped leader has approached within 15 to 50 meters of the ground, the electric field intensity at the ground becomes sufficient enough to initiate one or more positive leaders which will propagate upwards to meet up with the oncoming stepped leader. In this study, DB model was extended to simulate the process of positive leader growth.

Propagation of bidirectional discharges: Original DB model introduced by Niemeyer et al. [1] only allows unidirectional discharge propagation. However, the DB model developed in this study implicitly allows the propagation of both negative and positive discharges. The obvious reason for this behaviour is that the probability of growth is related to the *magnitude* of the local electric field. Therefore, independent of the type of the discharge, the pattern will tend to develop in the directions with higher potential gradient. That is, negative discharges (which carry negative charge and have lower potential) tend to propagate towards regions of higher potential while positive discharges (which carry positive charge and have higher potential) tend to propagate towards regions of lower potential.

When generating negative discharges, discharge initiation points were chosen from the lattice points comprising the cathode, and the initial set of possible breakdown links were extended from those chosen points. (In lightning configuration, the central point of the upper face of the lattice was chosen as the only discharge initiation point.). Similarly in order to simulate positive discharges, one or more lattice points comprising the anode have to be designated as discharge initiation points, and initial set of possible breakdown links have to be extended from those points. Once the initial set of possible breakdown links are specified accordingly, positive discharges will propagate in the same step-by-step manner as the negative discharges.

Positive leader initiation phase: In the original DB model, discharge initiation phase is not explicitly modelled. During the first iteration of the algorithm, it is assumed that the conditions for discharge initiation have already been satisfied by the initial set of possible breakdown links. However in order to simulate the development of positive leaders, an explicit initiation condition for positive discharges had to be enforced.

Figure 2 (a) shows the initial state of a 2D sample lattice. All lattice points on the ground layer (anode) are connected to adjacent charge-free lattice points on the layer above to form the initial set of possible breakdown links for positive discharges (indicated by grey colour dashed lines).

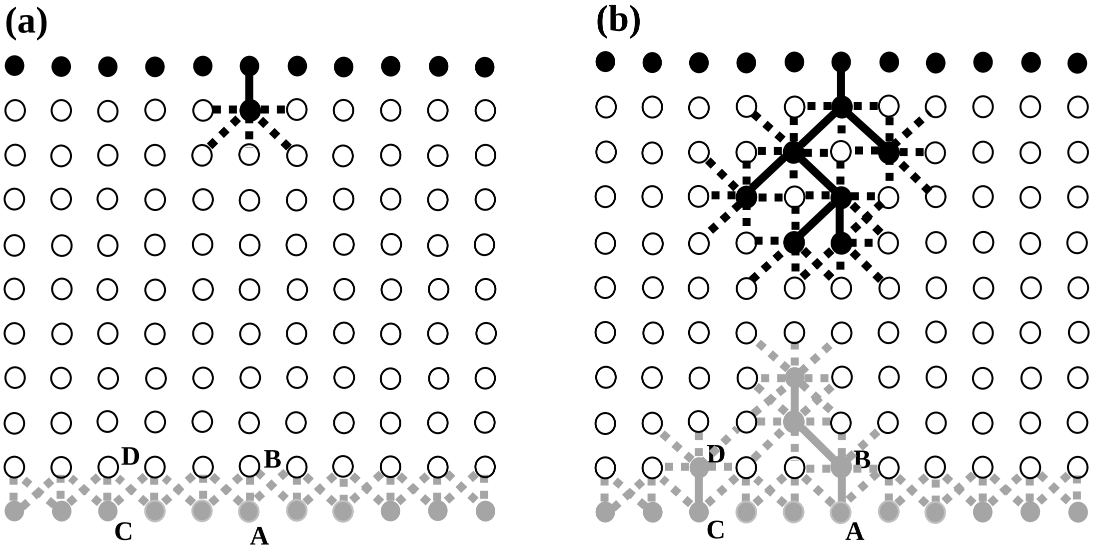

Figure 2: A sample 2D lattice showing positive leader development. (a) Initial charge configuration (b) Configuration after several iterations

However, not all those links are capable of being selected as a discharge link. For an initial possible breakdown link to be chosen as a discharge link, local electric field associated with the link must exceed a threshold value E_{break}.

During each iteration, the algorithm scans through the initial set of possible breakdown links to check whether the local electric field associated with any of them have exceeded E_{break}. If such links exist, one of them will be chosen according to equation 3 and added as the first discharge link of a new positive leader (e.g. AB). After the first discharge link of a new positive leader is formed, that leader is allowed to develop according to the same propagation rules as the negative discharge (Threshold E_{break} only influence the selection of *initial* possible breakdown links. Possible breakdown links formed later in the process are only subjected to the selection rule given in equation 3).

3. RESULTS AND DISCUSSION

The exponent η in equation 3 parameterize the relationship between the local electric field and the probability of growth of the discharge pattern. The overall appearance of the pattern is strongly related to η. Figure 3 shows two lightning discharge patterns for different η values. It can be seen that as η is increased, discharge patterns become sparser with reduced side branching while for lower values of η, discharge patterns become "bush" type densely packed patterns.

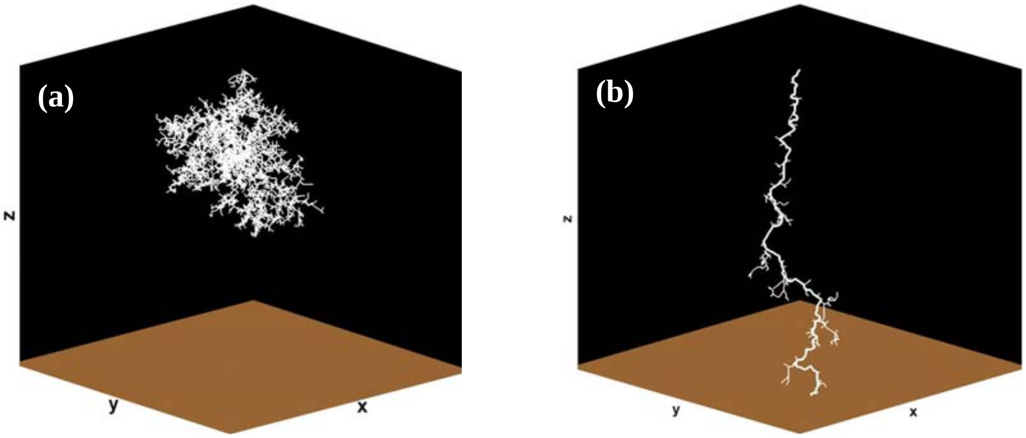

Figure 3: Simulated 3D lightning discharge patterns when (a) η=1 and (b) η=6

3.1. Influence of ground objects on lightning flashes

One of the most important features of DB model is that it allows the use of attractors and repulses to direct the growth of the discharge pattern. The growth can be attracted to a user specified region by setting the boundary condition $\phi = 1$ for the lattice points occupied by that region, while the growth can be repulsed from the region by setting $\phi = 0$. In this study, additional attractors were introduced into the 3D lightning configuration to simulate the effect of objects on the ground plane during lightning flashes.

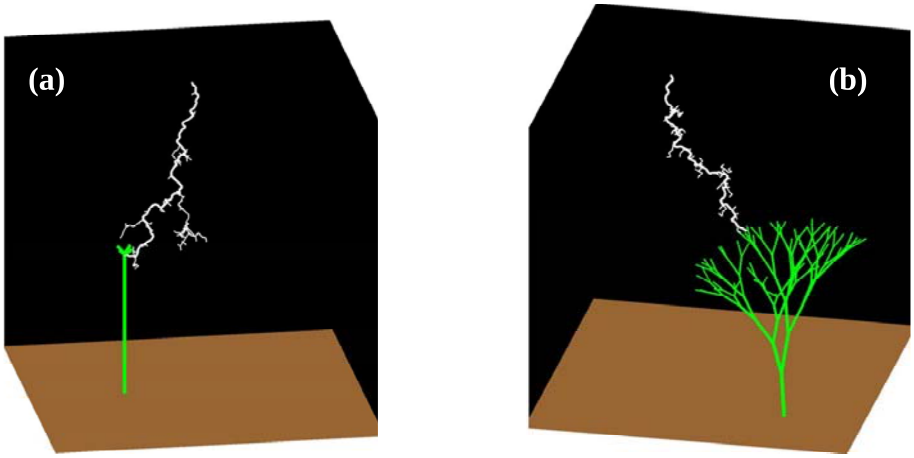

Figure 4: Simulated lightning discharges striking objects on the ground. (a) A simple vertical rod (b) A complex tree-like structure

Figure 4 shows final visual output of two simulations conducted to demonstrate the behaviour of simulated lightning discharges near such ground objects. Additional boundary conditions imposed by these structures were incorporated by setting the potential of the lattice points occupied by the grounded structures to $\phi = 1$. In both simulations, lightning discharge was attracted towards the grounded structure, instead of the ground plane. The obvious reason behind such behaviour is that the local electric field near the pointed surfaces of the structures is relatively higher than that of the ground plane. Since the growth probability of the discharge is dependent on local electric field, propagation of the pattern gets oriented towards those areas of high local electric field.

Due to its stochastic nature, the exact paths taken by real lightning flashes are unpredictable. Although taller pointed structures on the ground have a higher probability to be struck by lightning, predicting whether CG flashes initiated from a nearby cloud base will hit the structure is not possible. In order to justify that the same stochastic property is conserved in DBM simulations as well, a simple analysis was conducted using the grounded vertical rod shown in Figure 4 (a).

By varying the height (*h*) of the rod and the horizontal distance (*d*) from the discharge initiation point, a set of simulations were carried out to determine the frequency of lightning strikes. Twenty simulations were carried out for each combination of *h* and *d*, and the number of times the rod was struck by lightning was counted for each case. The obtained results are given in Table 1 (All lengths and heights are given in terms of the nearest neighbour distance of the lattice).

Table 1: Frequency of lightning strikes on the rod for various values of *h* and *d*

h	*d*	No of strikes (out of 20)	Probability of lightning strike
45	20	15	0.75
40	20	12	0.60
30	20	10	0.50
30	30	6	0.30
30	35	4	0.20

Table 1 proves that simulated lightning flash hitting the rod is not a deterministic event. For every combination of *h* and *d* considered, probability of a lightning strike is less than 1; that is there's always a non zero probability for the simulated lightning flash to miss the rod and hit the ground nearby. However, probability of striking the rod seems to be dependent on the height (h) of the rod and the horizontal distance (*d*) from the discharge initiation point. Probability of a strike seems to increase with height of the rod. This observation agrees with the situation regarding real lightning strikes; that the probability of taller objects getting hit by lightning is higher than that of shorter objects. Also it can be noticed that the probability of a strike decreases as the distance to the discharge initiation point (*d*) is increased; which is also an obvious situation one can observe with real lightning flashes.

3.2. Positive leader development

Although the breakdown physics are different for negative stepped leaders and positive leaders, for simplicity, both types of discharges were treated in the same way in this simulation work. That is, the value of η used to control stepped leader growth ($\eta = 6$) was used to control positive leader growth as well. However it should be noted that one can change the growth and appearance of positive leaders by simply using a different value of η for positive leaders.

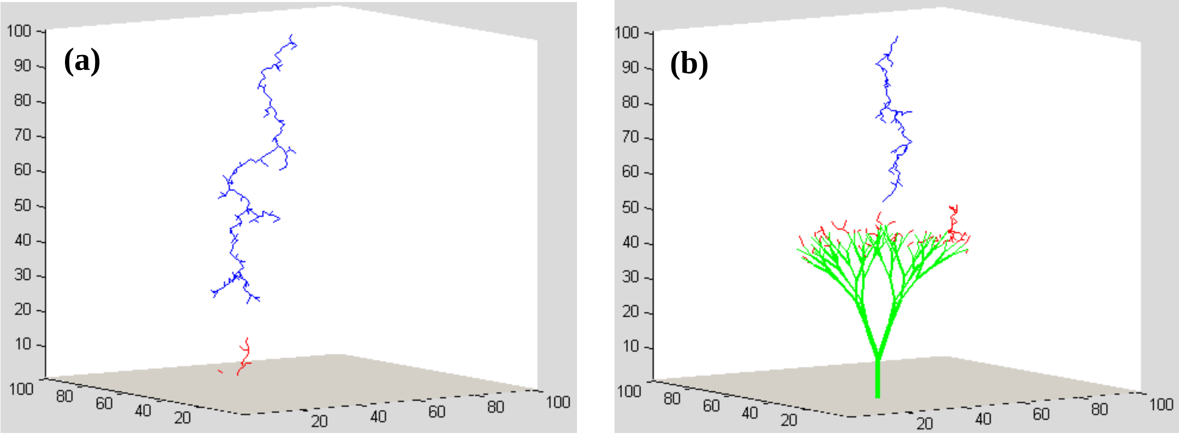

Figure 5: Positive leader development (a) Leaders developing from earth surface (b) Leaders developing from the tips of the tree-like structure

Figure 5 (a) shows a snapshot taken from a simulation conducted to demonstrate positive leader development from the ground plane. The negative stepped leader is indicated in blue while the positive leaders shown in red. At the time when the snapshot was taken, two leaders appear to have developed from the ground plane. The threshold for positive leader initiation was set to $E_{break} = 0.02$. The first positive leader initiated from the ground, when the tip of the stepped leader approached at a height of 23 units from the ground. At a height of 17 units from the ground, the longest positive leader connected with the stepped leader, forming the complete lightning path.

It was speculated that the threshold E_{break} determines the height to the stepped leader tip from the ground, when the positive leader initiation occurs. In order to test this hypothesis, E_{break} was varied from 0.02 to 0.07, and for each case, height of the leader tip above the ground (H) at the time when the first positive leader initiated was determined. Results clearly showed that the average height $\overline{H}$ exponentially decreases with E_{break}. This behaviour is quit logical since the role of E_{break} is to suppress the effect of local electric field when selecting initial discharge links for new positive leaders. As the stepped leader approaches at a certain height from the ground, local electric field associated with one or more possible breakdown links (that are relevant to positive leader initiation) will exceed E_{break},, initiating positive leaders. But if E_{break} is increased, a much higher value of local electric field may be needed to surpass it, and the stepped leader may need to descend further down to produce that same effect.

Figure 5 (b) shows a snapshot taken from another simulation conducted to demonstrate positive leader development in an environment with a complex tree-like structure [4] on the ground. E_{break} was set to 0.04, and the positive leader initiation occurred (from a tip of the tree), when the tip of the stepped leader approached at a height of 78 units from the ground. Figure 5 (b) shows that at the time when the snapshot was taken, positive leaders have developed from most of the tips of the tree. The obvious reason for this behaviour is that the local electric field near the tips is much greater than the surrounding regions, which allows more leaders to be initiated from the tips.

4. CONCLUSIONS

In this work, stochastic dielectric breakdown model was applied to 3D Cartesian geometry to simulate CG lightning flashes. By introducing additional boundary conditions into DB model, influence of ground objects on lightning flashes can be simulated. Tall, pointed structures on the ground have a higher probability of attracting simulated lightning discharges. Probability of strikes increases with the height of the ground object while it decreases as the horizontal distance to the discharge initiation point is increased. Novel extension introduced into the DB Model can successfully simulate the development of positive leaders during CG lightning flashes. Height of the stepped leader tip above the ground (at the time when the positive leader initiation occurs) is dependent on the newly introduced parameter E_{break}. Average height to the stepped leader tip decreases exponentially as E_{break} increases.

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